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ABSTRACT

Errors, or "bugs", in addition and subtraction problems of signed numbers are driven by misconceptions or incomplete knowledge at various components of procedural tasks, and result in erroneous rules of operation for carrying out computational problems. These rules, including the right one, are represented by the responses to a particular set of items because each rule generates a unique set of responses to the items. This study attempts to develop a method for quantifying these bugs according to the seriousness of the mistakes. This is done by referring to the procedural network, or task tree. If a role is a result of a misconception of an earlier level in the network, then it is more likely committed by students who are in the early stage of learning, or by lower ability students. The Procedural Steps Conformity Index was designed to quantitatively express these errors. (Author/BW)

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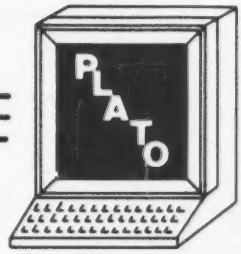

University of Illinois Urbana Illinois

AN APPROACH TO ASSESSING THE SERIOUSNESS OF ERROR TYPES AND PREDICTABILITY OF FUTURE PERFORMANCE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Brown & Burton found 108 bugs in whole-number subtraction problems. Tatsuoka et. al., found 57 bugs in signed-number arithmetic. This study attempts to develop a method for quantifying these numerous bugs according to the seriousness of the mistakes. This is done by referring to the procedural tasks and classifying them in terms of easy-and-harder-to-remove errors. The seriousness of the errors is discussed

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in connection with the predictability of the future performance and the amount of additional instructions and review which must be given to students in order to correct their misconceptions.

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Louise Brodie typed the report and drew the figures.

INTRODUCTION

Birenbaum & Tatsuoka (1980) and Tatsuoka, et. al. (1980) found quite a number of "bugs", 57, in addition and subtraction problems of signed numbers. These bugs are driven by misconceptions or incomplete knowledge at various components of procedural tasks and resulted in erroneous rules of operation for carrying out computational problems.

These rules, including the right one, are represented by the responses to a particular set of items because each rule generates a unique set of responses to the items. For example, the rule that adds the absolute values of two numbers and takes the sign of the first number in the answer yields responses +15, -15, and -19 to the problems, 12 + -3, -3 + 12 and -14 + -5, respectively. But the right rule produces responses +9, +9 and -19 to the same problems. By selecting an appropriate set of items, the two rules can be identified by examining the response vectors (+15, -15, -19) and (+9, +9, -19) respectively. Indeed, some error diagnoses are derived by human intuition from the examination of responses.

Brown & Burton (1978) have developed computerized diagnostic models of procedural skills in arithmetic computations. Their models provide an identification of mistakes in the procedural network. Tatsuoka, et al. (1980) introduced an "error vector system" that has a power equivalent to the procedural-network approach. The error vector system begins with the decomposition of a computational task of two signed numbers into component procedures, obtaining the absolute values and obtaining signs for answers. Thus, the two subprocesses are represented by two error vectors, absolute value and sign error vectors, whose components correspond to all possible alternative operations.

The mechanism of diagnosing the misconceptions possessed by a student starts first by generating a binary vector of erroneous and correct actions taken by the student. Thus, several different binary vectors (according to the number of component procedures necessary for carrying out the task) will be generated from the student's responses to the items. In the case of signed-number arithmetic, the two binary vectors that represent "getting the absolute values" and "choosing the signs in the answers" will be generated. The matrix product of the two binary vectors will determine a consistent, complete error committed by the student, if any.

Glaser (1981, p. 10) states that "An important skill of teaching is to identify the nature of the concept or rule that the student is employing that governs his performance in some systematic way, (in most cases, the student's behavior is not random or careless, but is driven by some underlying misconceptions or by incomplete knowledge)." The same results were observed by a series of experimental data conducted by the author (1981).

When the student applies his/her rule consistently for obtaining absolute values but inconsistently for taking in the signs of the

answers, then incomplete rules will be the outcome of error analysis. Birenbaum (1981) lists all the erroneous rules she found by examining students' response sequences. Chaiklin & Greeno (1981) describe cognitive backgrounds for these rules. Neither the computerized approaches of Brown & Burton (1978) nor that of Tatsuoka, et al. (1980) can diagnose such incomplete rules; in other words, inconsistent responses in at least one of the procedural components makes their task difficult.

Some rules may be incomplete, but, as long as they are consistent, their nature of error can be characterized by assessing the "seriousness". Some errors are due to misconceptions originating at the basic understanding of negative numbers, while other errors are caused by minor mistakes. The former type of error may be harder to correct, while the latter case will be easy to correct. It can be said that the serious errors are caused by misconceptions originating at the upper-level nodes in the procedural network, if procedural components are hierarchically related.

This study describes an attempt to quantify the degree of "seriousness" of various error types (or erroneous rules of operations) and investigates their relationships to the future performance of the student. The development of a methodological tool will be discussed in conjunction with the error analysis performed on signed-number addition problems. The method will be applicable to other arithmetic, mathematics and science problems.

CLASSIFICATION OF RULES

Complete and Incomplete Rules

Procedural representation of solving a given problem, such as subtraction problems in whole numbers (Brown & Burton, 1978), addition of two fractions (Greeno, 1976) and signed-number arithmetic (Birenbaum & Tatsuoka, 1980), has become a popular way to express the complex relationships among various procedural components. In order to achieve the goal, one must perform each subprocedure correctly and go to the next component. All necessary procedural steps must be completed successfully. If a student fails to follow a right procedural step, and does not perform all subprocedures correctly, then the outcome is likely an erroneous rule of operation or some mixture of erroneous rules.

Tatsuoka et al. (1980) developed a rule-diagnostic model, termed the "error vector system" for signed-number addition and subtraction problems. They decomposed an operation of computing these problems into two component procedures and represented each component procedure by an error vector: one for absolute value and the other for sign. The elements of the two vectors are prospective erroneous rules of operation that may be obtained by a close examination of component procedures. Descriptions of the rules are given in Table 1.

Insert Table 1 about here

As can be seen in Table 1, the absolute value vector is composed of seven elements, and the sign vector of nine.

For example, suppose a student subtracted the two numbers in the task, -5 + -3, and took the sign of the larger number in the answer. Then this erroneous rule of operation is expressed by (13,21). The right rule of operation is designated by (11,21) for addition problems.

The matrix product of the two error vectors will have 63 events that correspond to different rules of operation. In general, there are several error vectors, each of which represents a component procedure. A cartesian product of these error vectors will determine various erroneous rules of operation.

A rule is said to be complete if all component procedures are consistently performed by some systematic behavior. In other words, the responses to the items generated by a complete rule will be uniquely determined in the cartesian space of the error vectors. If at least one of the component procedures is not consistent then a rule is said to be incomplete. An incomplete rule cannot produce a unique set of responses to the items. One of the popular errors in signed-number arithmetic is "taking the right signs in the answers but absolute values are taken inconsistently." With such a partially right rule, the diagnostic models cannot determine and state the rule completely. Table 1 provides the error code (21) with this incomplete rule. Similarly other incomplete rules will be designated by two-digit numbers between 11 and 19 or 21 and 29 according to which component procedure is consistent.

Table 2 summarizes the number of complete and incomplete rules recovered by the error-vector system for a 20-item computation test. Although Table 2 shows the erroneous rules for only addition problems, those of subtraction problems have similar results (K. Tatsuoka, 1981).

Insert Table 2 about here

The test was administered to the seventh graders at a local junior high. About 70% of 290 sets of responses to five addition problems are successfully diagnosed by the error-vector system. The numbers of complete and incomplete rules are about the same, but the distribution of frequencies over the errors is far from uniform. Fifty-five sets of responses to five addition items demonstrated the right rule (11,21), 20 and 14 response sets are yielded by the incomplete right rules (11) and (21), respectively. The incomplete rules (13) and (24) have frequencies of 24 and 23, which are about 8% of the total 290 response sets. Other erroneous rules such as (12) and (22) are rare.

Investigation on the differences in the frequencies of error types may be very important in understanding students' states of learning and improving the quality of teaching.

Table 1

Decoding of the Basic "Error-Vector System" of Signed-Number Operations for Two Numbers

addition problems such as

$-L+S$ (*assume $|L| \geq |S|$

Error Type		Description of the operation
Absolute Value operations	11	If the signs of two numbers are same, then $\|L\|+\|S\|$ If the signs of two numbers are different, then $\|L\|-\|S\|$
	12	Always $\|L\|+\|S\|$, adding the absolute values of L, S.
	13	Always $\|L\|-\|S\|$
	14	Opposite of the right operation
	15	If the sign of the first number is +, then $\|L\|+\|S\|$ If the sign of the first number is −, then $\|L\|+\|S\|$
	16	If the sign of the second number is +, then $\|L\|+\|S\|$ If the sign of the second number is −, then $\|L\|-\|S\|$
	17	If the sign of the first number is + then $\|L\|-\|S\|$ Otherwise opposite
Taking the sign of answers	21	Sign of the larger number
	22	Sign of the smaller number
	23	Always +
	24	Always −
	25	Sign of the first number
	26	Sign of the second number
	27	Sign of the product of the two numbers
	28	Change the sign of the second number, then apply 21
	29	Change the sign of the first number, then apply 21

Table 2

Frequencies and Error Types Recovered by the Error-Vector System in Test1, Addition Problems of Signed Numbers

Incomplete Rule	Frequency	Complete Rule	Frequency
11	20	11,21	55
12	2	12,23	3
13	24	12,24	5
21	14	12,25	3
22	3	13,21	17
23	15	13,22	1
24	23	13,23	5
25	6	13,24	10
26	5	13,25	3
16	1	13,25	3
Total*	113	Total	105

*72 Response Patterns are either inconsistent or generated by rules besides those in Table 1

ASSESSING THE SERIOUSNESS OF ERRORS

Are Two Rules Independent?

Two different rules may represent different dimensions and hence may be statistically independent. But others might not be independent from one another. Of course, errors are outcomes of misconceptions or incomplete knowledge somewhere in a series of complex procedures. If a student does not understand the difference between operation signs and the signs of numbers, then it is very unlikely that his/her errors are derived by certain systematic mistakes such as "taking the sign of the first number in his/her answer." His/her erroneous rule will more likely by derived from an ignorance of the concept new to signed-number arithmetic, so it may be like "the sign of the answer is always minus for subtraction and plus for addition problems." These two error types will probably not appear together in the responses by a single student to the test items.

Table 3 shows the list of rules committed by the students which are obtained from the error analysis on the same test described earlier.

Insert Table 3 about here

The 52-item test was designed to examine stability of erroneous rules of operation so that four 13-item, parallel subtests are repeatedly given in the test. The rule in the first pair of parentheses stands for the first subtest, and the fourth for the fourth subtest in the 52-item test. Error patterns are examined separately for addition problems and subtraction problems in each subtest. Table 3 provides information obtained from analysis of addition probems. The Greek letter θ in the last column of the table denotes a student whose verbal ability is high, within the top 16% of the students.

Insert Table 4 about here

Since each student took the four parallel subtests, the frequency tables of rules for the first and second subtests are obtained separately and χ^2 - tests for independence of rules are carried out. The first test has χ^2 - value of 22.57 with $p = 0.0002$ and the second test has $\chi^2 = 18.62$ with $p = 0.0009$. The results suggest that the rules are not statistically independent. Thus, the students who have mastered the right operation in terms of the absolute-value operation tend to achieve the right operation for taking the signs in their answers. Those who answered inconsistently with regard to taking the signs in the answers also have a tendency toward having inconsistent absolute value operations.

A close investigation of changes in error types over the four subtests reveals that there are apparent systematic relationships in shifting to one rule from another, although quite a number of students repeatedly used

Table 3

List of Error Types in the First Test of the January Experiment
for those Students Whose Responses are Highly Consistent

Subject	Error Type	Score	Subject	Error Type	Score
40	(11,21)(11,21)(11,21)(11,21)	19	3	(11,21)(11,21)(11,21)(11,21)	20
52	(21)(11,21)(11,21)(11,21)	19 *	4	(11,21)(11,21)(11,21)(11,21)	"
17	(11,21)(13,21)(21)(11,21)	18 θ	13	(11,21)(11,21)(11,21)(11,21)	"
79	(13,21)(11,21)(11,21)(11)	18 θ	34	(11,21)(11,21)(11,21)(11,21)	"
12	(11)(21)(11,21) 0 +	16 θ	65	(11,21)(11,21)(11,21)(11,21)	20
45	(13,21)(11,21) 0 (21)	16 θ	27	(11,21)(11)(11,21)(11,21)	19
55	(11)(11,21)(21)(11)	16	35	(11)(11,21)(11,21)(11,21)	19
31	(11)(13)(13,21)(11,21)	15 θ	36	(11,21)(11)(11,21)(11,21)	19
44	(11,21)(24)(21)(13,21)	15	41	(13,21)(11,21)(11,21)(11,21)	19
73	(13)(11,21)(11)(11)	15	42	(13,21)(11,21)(11,21)(11,21)	19
17	(11)(13,21)(13)(13)	14	58	(13,21)(11,21)(21)(11,21)	18
20	(13,21)(21) 0 (13,21)	14	37	(13)(11,21)(11)(11)	17
70	(21) 0 (21) 0	14 θ	50	(13,21) 0 (11,21)(11,21)	17
72	(17,21)(17,21) 0 0	14 θ	29	(13,21)(13,21)(21)(13)	14
30	0 (21) 0 (13,21)	13 θ	53	(11,21)(11,21) 0 0	14
60	(11)(13) 0 0	13 θ	2	0 (25)(13,25)(13,25)	10
57	0 (13)(13,21)(13)	12 θ	22	0 (13,24)(13)(13)	10
18	(13) 0 (13)(13,23)	10	48	(13,21)(13,23)(13,23)(13,23)	10
10	0 0 (13) 0	9	46	0 0 (13,23)(23)	9
23	(24) 0 (13,24)(24)	9	56	(11)(21)(23) 0	9
1	0 (13,24)(13,24)(24)	8	26	(25)(13,25)(13,25) 0	8
7	(13,21) 0 0 0	8 θ	76	(23)(13,23)(13,23)(13,23)	8
8	(13)(13)(13)(13)	8	16	0 (26) 0 0	7
54	(13,24) 0 (21)(25)	8	43	0 (13)(13)(13)(13)	7
59	0 0 0 0	7	24	(16,26)(16,26)(16,26)(16,26)	5
5	0 (16,23) 0 (16,25)	6	21	(24)(24)(12,25)(12,25)	4
67	(22)(23) 0 0	6	25	0 0 0 0	4
78	(24)(24)(24)(24)	6	49	(12)(12)(12,24)(12,24)	4
9	(13,24)(24)(12,24)(12,24)	4 θ	64	0 (12,23)(12,23) 0	1
47	(22)(13)(23)(13)	4	62	0 (24)(24)(24)	2
14	0 0 0 0	3	71	0 0 (24) 0	2
69	0 0 0 (12,23)	3	6	0 (21) 0 0	1
19	(23)(12,23)(23)(12,23)	2	39	(24)(24)(24)(12,24)	2

*Verbal ability estimates of larger than 1.0

+Inconsistent responses

Table 4

Test for Independence over Erroneous Rules of Operation

Addition Problems

Subtest 1

Rules \ Rules	(21)	(24)	Inconsistent	
(11)	17	0	3	20
(13)	3	3	6	12
Inconsistent	4	6	13	23
	24	9	22	55

$X^2 = 22.57$ $p = 0.0002$

Subtest 2

Rules \ Rules	(21)	(24)	Inconsistent	
(11)	11	0	7	18
(13)	10	3	4	17
Inconsistent	2	3	18	23
	23	6	29	58

$X^2 = 18.62$ $p = 0.0009$

Subtest 3

Rules \ Rules	(21)	(24)	Inconsistent	
(11)	15	0	2	17
(13)	2	4	6	12
Inconsistent	7	4	15	26
	24	8	23	55

$X^2 = 22.24$ $p = 0.0002$

Subtest 4

Rules \ Rules	(21)	(24)	Inconsistent	
(11)	16	0	4	20
(13)	3	1	7	11
Inconsistent	1	4	17	22
	20	5	28	53

$X^2 = 26.69$ $p < 0.00005$

their own rule consistently in order to respond to the items across the four subsets in the test. Incomplete and complete rules often appear together when the former is a partial rule of the latter. For example, the pattern of student 1, 0 (13,24)(13,24)(24), is a typical example of the situation. The second and third are complete rules, while the fourth is an incomplete rule. This student's state of learning is obviously around the wrong rule (13,24). His rule is always to take the difference of absolute values of the two numbers, the smaller one from the larger one, and using minus signs in the answers. He did not apply an appropriate rule of operation for different types of problems and also failed to discriminate the operation sign of the problems from the signs of the numbers. It is an understandable mistake because he has not yet studied subtraction skills when he took the test.

The second kind of pattern in Table 3 is that of incomplete and complete rules appearing together but the incomplete one is not a partial rule of the complete rule. For example, student 54 has the pattern, (13,24) 0 (21)(25). The rule related to the sign operation changes three times: 24, 21 and 25, that is, "always taking a minus sign", "taking the sign of the larger number" and "taking the sign of the first number". It can be said that this student is not confident with her rule of taking the signs in answers, so she chooses the signs arbitrarily by trial and error. Her stability over the four subtests is low. Thus it is difficult to locate her rule from the test, at least from the present information about her performance on the test. It might be necessary to give her more items so as to determine her rule.

The third kind of pattern is expressed by the following example: students 41, 42 and 58 always subtracted two absolute values and took the sign of the larger number in the answers. Their pattern is (13,21)(11,21)(11,21)(11,21). Their rules reached the right rule of operation at the second try. The change of rules involves two complete rules, (13,21) and (11,21).

The fourth kind includes 0 , which means the error vector system could not identify the rules used by students. So we characterize the rules as "inconsistent rules" as far as they can not be determined by the present error-vector system.

The errors resulting from misunderstanding one or two basic concepts can be said to be serious because it is not easy to remove these errors. The students have to study the basic concepts which are usually presented at an earlier part of instruction. The third kind of error type, switching from (13,21) to (11,21) might be easy to correct because (13,21) came from ignoring the difference between skills of the -L + -S and -L + S types. If the two signs of the numbers are the same, the absolute value for answers must be obtained by adding two numbers instead of subtracting them. The student knows the difference between operation signs and the sign of a number. It can be said that he/she just failed to distinguish a -L + -S type skill from a -L + S type skill. Indeed, (13,21) and (11,21) appeared together in one response

pattern quite often (11 times). In this sense (13,21) is not serious and is easy to correct.

QUANTITATIVELY ASSESSING THE SERIOUSNESS OF ERRORS

Definition of Seriousness

One approach to quantifying the seriousness of error types is to assess the extent to which a rule vitiates the steps necessary to achieve a given task successfully. In other words, each error must have reasons as to why and where it occurred. Some errors are due to students' careless mistakes, but others -- quite a number of them -- are due to the lack of understanding at some level of the procedural steps. If the errors were caused by misunderstanding at some deeper level of the procedure, then it will not be easy to remove them. If the errors were caused for two or more compounded reasons such as missing several procedural steps to arrive at the right answer, then this is more serious than an error caused by missing just one step. If the error occurred at steps close to the end of the procedural network, then this error might be easy to correct; thus, it is less serious. In order to assess the seriousness of errors in terms of the procedural network, we must know the network first. Regarding the networks, since there is much good literature written by cognitive psychologists (e.g., Greeno, 1976; Resnick, 1976; Greeno, 1976), the author assumes that a suitable task analysis has already been achieved and hence we know our network.

Suppose everybody knows what negative numbers are and can discriminate them from positive numbers. Thus, knowing the negative and positive numbers is required as a prerequisite of the tasks of carrying out the operation of addition problems. Then the next step is that students must discriminate operation signs from the signs of numbers. The third step is the identification of the type of addition task, +[ ] + -[ ], -[ ] + [ ] or [ ] + [ ], -[ ] + -[ ]. For tasks in which the two numbers have different signs, the right absolute values of answers are gotten by subtracting the absolute value of the smaller number from the larger number, while the other tasks require adding the absolute values of the two numbers. After taking the right operation of absolute values, the students have to choose the right sign for their answers, which involves taking the sign of the larger number in both cases.

By representing all component procedures for carrying out the addition problems properly, Figure 1 gives a graphical representation of the procedural network. Note that the number at the upper left corner of the rectangle for each node represents the level of that task in the hierarchy.

Insert Figure 1 about here

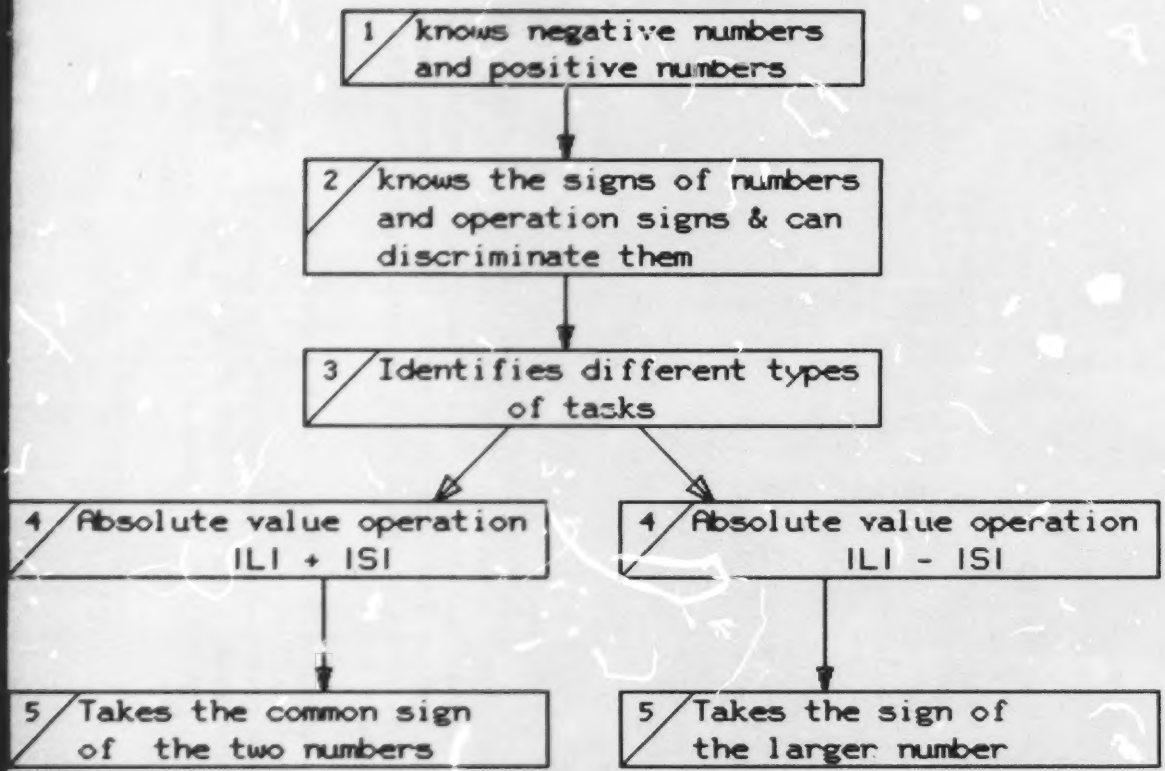
1 knows negative numbers and positive numbers
2 knows the signs of numbers and operation signs & can discriminate them
3 Identifies different types of tasks
4 Absolute value operation |L| + |S|
4 Absolute value operation |L| - |S|
5 Takes the common sign of the two numbers
5 Takes the sign of the larger number

Figure 1: Task Tree of Signed-number Addition Problems

Procedural Steps Conformity Index (PSCI)

Tatsuoka & Tatsuoka, (1980) introduced a new index, the Norm Conformity Index, which measures the degree of conformity of a response pattern to a criterion order for each examinee. The criterion order is usually determined by some external consideration such as adopting the order of item difficulty in a test administered to a certain group, or that of a national norm group. We now conider possible orderings of component procedures instead of items. Since the base order is again arbitrary, the order determined by the component hierarchy is one of natural choice. The sequence from 1 through 5 in Figure 1 will be taken as the base order. These procedures thus ordered will be referred to as procedural levels hereafter.

A rule can be checked with each procedural level to see whether or not it is based on a misconception or incomplete knowledge occurring at that level. By assigning 1 or 0 to all the procedural levels, the rule will be represented by a binary vector. The vector will be referred to as a process pattern hereafter.

For example, if the item -14 + -5 has been responded to by the rule (13,21), the process pattern will be (11001), by the following reasoning. The first element is assumed to be 1 for the relevant grade. Since incomplete rule 21 implies that a student knows the difference between the sign of the numbers and the operation sign, the second element of the process pattern will be 1. Using partial rule 13 (always doing $|L| - |S|$) implies inability to identify the task type; hence the third and fourth elements of the process pattern are 0. The fifth element is obviously 1.

The test administered to the seventh graders contains five addition tasks: (1) L + -S, (2) -S + L, (3) -L + -S, (4) S + -L and (5) -L + S. For each of these five tasks a process pattern is defined by any given rule of operation. Those corresponding fifteen of the erroneous rules obtainable from Table 1 are summarized in Table 6.

Insert Table 6 about here

Procedural Steps Conformity Index (PSCI) is defined as a linear transform of the following ratio: the number of transpositions needed to transform a process pattern vector to its reverse Guttman scale, divided by the total number of possible transpositions. Let us denote the former by U_a and the latter by U. Then:

$$PSCI = 1 - 2U_a/U$$

The total number of 1's in a process pattern shows the number of procedural levels a student followed correctly. The average of the

Table 6

Process Pattern Vectors Associated with Task Components for Five Item Types, 1. L+-S, 2. -S+L, 3. -L+-S, 4. S+-L and 5. -L+S.

Error Types		(11)	(12)	(13)	(11,21)	(12,21)
Component	Tasks	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
1		1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1
2		1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	1 1 1 1 1	1 1 1 1 1
3		1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	1 1 1 1 1	1 1 1 1 1
4		1 1 1 1 1	0 0 1 0 0	1 1 0 1 1	1 1 1 1 1	0 0 1 0 0
5		0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	1 1 1 1 1	1 1 1 1 1
Total 1's		4 4 4 4 4	1 1 2 1 1	2 2 1 2 2	5 5 5 5 5	4 4 5 4 4
PSCI		1 1 1 1 1	1 1 * 1 1	* * 1 * *	1 1 1 1 1	5 5 1 5 5

Error Types		(26)	(21)	(23)	(24)	(25)
Component	Tasks	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
1		1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1
2		0 0 0 0 0	1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
3		0 0 0 0 0	1 1 1 1 1	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
4		0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
5		0 1 1 1 0	1 1 1 1 1	0 0 0 0 0	0 0 1 0 0	1 0 1 0 1
Total 1's		1 2 2 2 1	4 4 4 4 4	1 1 1 1 1	1 1 2 1 1	2 1 2 1 2
PSCI		1 0 0 0 1	5 5 5 5 5	1 1 1 1 1	1 1 0 1 1	0 1 0 1 0

Error Types		(12,23)	(12,24)	(12,25)	(14,23)	(13,24)
Component	Tasks	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
1		1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1
2		0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
3		0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
4		0 0 1 0 0	0 0 1 0 0	0 0 1 0 0	1 1 0 1 1	1 1 0 1 1
5		0 0 0 0 0	0 0 1 0 0	1 0 1 1 0	0 0 0 0 0	0 0 1 0 0
Total 1's		1 1 2 1 1	1 1 3 1 1	2 1 3 2 1	2 2 1 2 2	2 2 2 2 2
PSCI		1 1 * 1 1	1 1 -* 1 1	0 1 -* 0 1	* * 1 * *	* * 0 * *

* = .33

total number of 1's across the five tasks will be called the mean procedural score (MPS) associated with a rule. The MPS and the average of the five PSCI values for each rule are considered as indicators of the seriousness of the mistake.

Table 7 summarizes the values of these indices for the same examples as given in Table 6. Figure 2 is a plot of the erroneous rules of operation listed in Table 6.

Insert Table 7 about here

Insert Figure 2 about here

From Figure 2, it is obvious that the rules are classified into two groups, A and B. The members in Group A cluster at the upper right corner of the plane while those in Group B fall in the extreme left side of the plane. The right rule (11,21) is located at the top-right corner and it belongs to Group A. Therefore the rules in Group A have much higher proximity to the right rule than those in Group B. Some of the notable characteristics of the rules in Group A are that (1) they follow more procedural levels correctly than the others, and (2) they tend to have higher values of the Procedural Steps Conformity Index. Further, Table 6 indicates the rules in Group B missed both of the important concepts at levels 2 and 3 but the rules in Group A followed them correctly and missed both or either of levels 4 and 5.

Changes in Errors as Learning Stages Advance

The bugs classified in Group B originate in the misunderstanding or ignorance of one or more basic levels of the procedural task. A question arises whether or not these error types have certain relationships with verbal ability because the instruction given before the test used verbal stories, the Postman Stories (Davis, 1965), to teach signed-number computations. A second question is whether or not frequencies of these erroneous rules depend on the learning stages of the students. Which rules will be observed more often in data as the students approach their mastery level? The answers to these questions can be found only by empirical investigation.

Insert Table 8 about here

The Stanford Verbal Test was administered to the same subjects as those in Table 7 and Figure 1. Their ability measures were obtained by applying Item Response Theory (Lord, 1980; Lord & Novick, 1968). In Table 2, the students whose estimated ability level for the test were in the top 16% (which is $\theta \geq 1$) were marked by θ. Table 8 indicates that 64% of the rules used by θ-students belong to Group A, and only 16% belong to Group B. The trend of percentages is reversed, however, for

Table 7

The Summary of the Means of the Number of Steps Followed by each Error Type in the Test Tree and of the Values of Process Tree Conformity Indices

Error Types	Mean of Steps	Mean of PTCI
(11)	4.0	1.00
(12)	1.2	.87
(13)	1.8	.47
(21)	4.0	.50
(22)	3.2	.90
(23)	1.0	1.00
(24)	1.2	.80
(25)	1.6	.40
(26)	1.6	.40
(11,21)	5.0	1.00
(11,22)	4.2	1.00
(11,24)	3.4	1.00
(11,25)	4.6	1.00
(11,26)	4.6	1.00
(12,21)	4.2	0.60
(12,22)	3.4	1.00
(12,23)	1.2	0.87
(12,24)	1.4	0.73
(12,25)	1.8	0.33
(12,26)	1.6	0.40
(13,21)	4.6	.87
(13,22)	4.0	0.70
(13,23)	1.8	0.47
(13,24)	2.0	.26
(13,25)	2.4	0.00
(13,26)	2.4	0.00

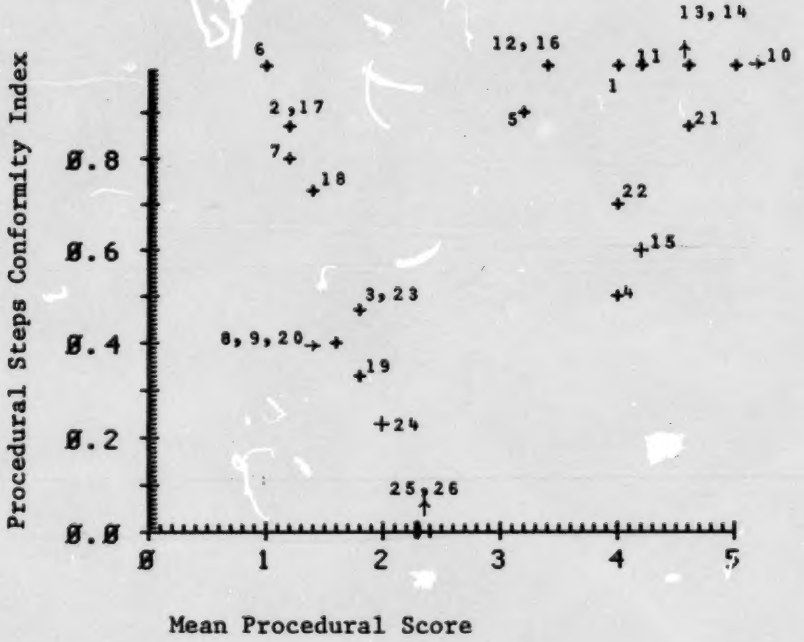

Figure 2: Graphical Presentation of 26 Rules by their Seriousness of Mistakes

Key for Figure 1

Number on the graph	Error Type
1	(11)
2	(12)
3	(13)
4	(21)
5	(22)
6	(23)
7	(24)
8	(25)
9	(26)
10	(11,21)
11	(11,22)
12	(11,24)
13	(11,25)
14	(11,26)
15	(12,21)
16	(12,22)
17	(12,23)
18	(12,24)
19	(12,25)
20	(12,26)
21	(13,21)
22	(13,22)
23	(13,23)
24	(13,24)
25	(13,25)
26	(13,26)

Table 8

The Ratio of Serious to Non-serious Errors as a Function of Verbal Abilities and Learning Stages

Error Types	Ability Class	January Data	November Data
		Postman Stories	
A*	$\theta \geq 1^{+}$	64%	
	$\theta < 1^{+}$	25%	
	all	34%	91%
B**	$\theta \geq 1$	16%	
	$\theta < 1$	38%	
	all	33%	1%

* Rules belonging to Group A; non-serious errors
** Rules belonging to Group B; serious errors
\+ $\theta \geq 1$ means students whose Stanford Vocabulary Test scores are in the top 15%, and $\theta < 1$ means the rest of the students

those students with lower verbal ability. Therefore, it can be said that less serious errors are observed among those having higher verbal ability while more serious errors were committed by lower verbal-ability students.

The November data shown in Table 8 were obtained from the same test as the test administered to the seventh graders (January data). The subjects in the November data were given the test after completion of three weeks of instruction by teachers, and the students performed very well on the test. But there is no information available on the verbal abilities of these students. 91% of the observed rules in the November subjects are in Group A and only 1% belong to Group B. Erroneous rules of operation were shifted from Group B to A as the learning stages of the students advanced. In the January data, where many students used a variety of rules consistently, rules in Group A were more often used among high verbal ability students.

DISCUSSION

Brown & Burton (1978), K. Tatsuoka, et al., (1980) and Brown & VanLehn (1980) have found many bugs in arithmetic skills of whole numbers and signed numbers. Although their approaches toward diagnosing misconceptions are not exactly the same, both models require a careful task analysis. Several researchers (Airasian, 1971; Bart & Airasian, 1973; Dayton, 1976; Price, 1978) are working toward constructing behavioral item hierarchies by using order analysis. Sato (1978, 1980) developed a method for constructing a task tree by using Graph Theory. (His work has been introduced in English by M. Tatsuoka (1979)). The method starts by making an adjacency matrix whose columns represent component procedures. After obtaining the reachability matrix from the adjacency matrix, Sato constructs a network. With his method, all procedural levels of a task are determined first, and then nodes representing the steps of the task are added to each level. The reachability matrix provides the direct and indirect relationships among the nodes in the network.

The seriousness of errors is determined by referring to the procedural network (or task tree). If a rule is the result of a misconception of an earlier level in the network, then it is more likely committed by students who are in the early stage of learning, or by lower ability students. When students are near the mastery level of performance, their erroneous rules (if any) are due to mistakes occurring at the end of the network. Some erroneous rules have resulted from misconceptions occurring at two or more levels and their sources of errors are compounded. PSCI was designed to quantitatively express these compounded error sources as well as those single error sources.

The example in this paper (addition of signed numbers) has an especially simple procedural representation, but our technique for assessing the seriousness of errors should be extendable to more general cases. For example, the method is now being tried out on signed-number

subtraction and fraction problems. Some networks might be too complicated for determining all the levels in a linear order. If several different sets of linearly ordered levels exist in a procedural network, further work will be needed for generalizing the definition of the Procedural Steps Conformity Index.

REFERENCES

Airasian, P.W. Validating sequential instructional hierarchies. Educational Technology, 1971, December, 54-56.

Bart, W.M., & Airasian, P.W. Determination of the ordering among seven Piagetian tasks by an ordering-theoretic method. Journal of Educational Psychology, 1974, 66, 277-284.

Birenbaum, M. Error analysis -- it does make a difference! (Doctoral dissertation, University of Illinois, 1981).

Birenbaum, M., & Tatsuoka, K.K. The use of information from wrong responses in measuring students' achievemnnt (Technical Report 80-1). Urbana, Ill.: University of Illinois, Computer-based Research Laboratory, 1980.

Brown, J.S., & Burton, R.R. Diagnostic models for procedural bugs in basic mathematics skills. Cognitive Science, 1978, 2, 155-192.

Brown, J.S., & VanLehn, K.V. Repair Theory: A generative theory of bugs in procedural skills. Palo Alto, Cal.: XEROX Palo Alto Research Center, 1980.

Chaiklin, S., & Greeno, J. The use of verbal rules in problem solving: A study of signed number arithemetic. Unpublished manuscript, 1981.

Dayton, C.M., & Macready, G.B. A probabilistic model for validation of behavioral hierarchies. Psychometrika, 1976, 41, 189-204.

Davis, R.B. The Madison project, discovery in mathematics; a text for teachers. Reading, Mass.: Addison-Wesley, 1964.

Glaser, R. The future of testing: A research agenda for cognitive psychology and psychometrics (Technical Report no. 3). Pittsburgh: University of Pittsburgh, Learning Research and Development Center, 1981.

Greeno, J.G. Cognitive objectives of instruction: Theory of knowledge for solving problems and answering questions. In D. Klahr (Ed.), Cognition and Instruction. Hillsdale, N.J.: John Wiley & Sons, 1976.

Gregg, L.W. Methods and models for task analysis in instructional design. In D. Klahr (Ed.), Cognition and Instruction. Hillsdale, N.J.: John Wiley & Sons, 1976.

Lord, F.M. Applications of item response theory to practical testing problems. Hillsdale, N.J.: Laurance Erlbaum Associates, 1980.

Lord, F.M., & Novick, M.R. Statistical theories of mental test scores. Reading, Mass.: Addison-Wesley, 1968.

Price, L.C. Algorithms for discovering hierarchical relationships among dichotomous items. Unpublished Doctoral dissertation, University of Maryland, 1978.

Resnick, L.B. Task analysis in instructional design: Some cases from mathematics. In D. Klahr (Ed.), Cognition and Instruction. Hillsdale, N.J.: ohn Wiley & Sons, 1976.

Sato, T. Determination of hierarchical networks of instructional units using interpretive structural modeling method. Japanese Journal of Educational Technology, 1979, 4(1),

Sato, T. Curriculum Design and Evaluation. Tokyo, Japan: Meiji Tosho, 1980.

Tatsuoka, K.K. The least-squares estimation of latent trait variables by a Hilbert space approach (CERL Report E-4). Urbana, Ill.: University of Illinois, Computer-based Education Research Laboratory, February, 1989.

Tatsuoka, K.K. Summary of automated error analysis for signed-number computations: Addition and subtraction (Research Report 81-4). Urbana, Ill.: University of Illinois, Computer-based Education Research Laboratory, 1981.

Tatsuoka, K.K., Birenbaum, M., Tatsuoka, M.M., & Baillie, R. A psychometric approach to error analysis on response patterns (Technical Report 80-3). Urbana, Ill.: University of Illinois, Computer-based Education Research Laboratory, 1980.

Tatsuoka, K.K., & Tatsuoka, M.M. Detection of aberrent response patterns and their effect on dimensionality (Technical Report 80-4). Urbana, Ill.: University of Illinois, Computer-based Education Laboratory, 1980.

Tatsuoka, M.M. Recent psychometric developments in Japan: Engineers tackle Educational measurement problems. ONR Tokyo Science Bulletin, 1979, 4, 1-7.

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